Decoding the Geometry of Heritage: Unveiling Affine Transformations in the Saoraja La Tenri Bali through Digital Reconstruction

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ARTIGO

Decodificando a Geometria do Patrimônio: Desvelando Transformações Afins no Saoraja La Tenri Bali através da Reconstrução Digital

Decoding the Geometry of Heritage: Unveiling Affine Transformations in the Saoraja La Tenri Bali through Digital Reconstruction

Resumo

A lógica matemática incorporada na arquitetura vernacular indonésia permanece subutilizada na educação matemática de graduação. Este estudo investiga o *Saoraja La Tenri Bali*, uma casa tradicional Bugis, como um contexto pedagógico para o aprendizado de transformações afins em álgebra linear. Baseando-se na etnomatemática, na "matemática congelada" e na Teoria da Casa Simples, empregamos a reconstrução digital (Blender) como uma abordagem de etnomodelagem para decodificar o conhecimento geométrico implícito da estrutura. A análise revela que o *Saoraja* é regido por transformações afins: seus 101 pilares seguem uma lógica de translação análoga aos algoritmos de matriz, enquanto o telhado depende de transformações de cisalhamento para estabilidade estrutural. Em uma atividade de graduação intercultural, os alunos reconstruíram o *Saoraja* usando transformações matriciais sob restrições geométricas. Esse processo exigiu distinguir entre operações de rotação, escala e cisalhamento com base em consequências geométricas, apoiando a compreensão conceitual além da manipulação simbólica. **Embora pesquisas anteriores tenham validado matematicamente a geometria deste artefato, o presente estudo é uma investigação independente focada na implementação pedagógica. Especificamente, a novidade pedagógica reside em demonstrar como tarefas de etnomodelagem digital baseadas em erros mediam a compreensão conceitual dos alunos sobre transformações afins.** Ao colocar em primeiro plano padrões de erros sistemáticos e revisão de modelos impulsionada por restrições, este estudo oferece uma abordagem pedagógica transferível para integrar artefatos culturalmente fundamentados no ensino avançado de matemática.

Palavras-chave: Etnomatemática. Transformações Afins. Etnomodelagem Digital. Educação Matemática no Ensino Superior. Arquitetura Bugis.

Abstract

The mathematical logic embedded in Indonesian vernacular architecture remains underutilized in undergraduate mathematics education. This study investigates the *Saoraja La Tenri Bali*, a Bugis traditional house, as a pedagogical context for learning affine transformations in linear algebra. Drawing upon ethnomathematics, "frozen mathematics," and Simple House Theory, we employ digital reconstruction (Blender) as an ethnomodeling approach to decode the structure's implicit geometric knowledge. Analysis reveals the *Saoraja* is governed by affine transformations: its 101 pillars follow translation logic analogous to array algorithms, while the roof relies on shear transformations for structural stability. In a cross-cultural undergraduate activity, students reconstructed the *Saoraja* using matrix transformations under geometric constraints. This process required distinguishing between rotation, scaling, and shear operations based on geometric consequences, supporting conceptual understanding beyond symbolic manipulation. **While previous research mathematically validated this artifact's geometry, the present study is an independent investigation focused on pedagogical implementation. Specifically, the pedagogical novelty lies in demonstrating how error-based digital ethnomodeling tasks mediate students' conceptual understanding of affine transformations.** By

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foregrounding systematic error patterns and constraint-driven model revision, this study offers a transferable pedagogical approach for integrating culturally grounded artifacts into advanced mathematics instruction.

Keywords: Ethnomathematics. Affine Transformations. Digital Ethnomodeling. Undergraduate Mathematics Education. Bugis Architecture.

1 Introduction

The traditional architecture of the Bugis people in South Sulawesi, Indonesia, is more than a shelter against the elements; it is a physical manifestation of a complex indigenous knowledge system. Among these architectural feats, the *Saoraja La Tenri Bali* (the Great House of La Tenri Bali) in the Wajo Regency stands as a monumental example. Characterized by its iconic 101 pillars and grand scale, this structure is not merely a cultural symbol but a repository of geometric intuition. For centuries, Bugis master builders have erected such structures without the aid of Cartesian coordinates or written matrix algebra, relying instead on embodied knowledge and cultural algorithms passed down through generations.

In the field of ethnomathematics, as defined by Ubiratan D'Ambrosio, mathematics is viewed as a cultural product—a set of practices used by specific cultural groups to explain and manage their reality (D'Ambrosio, 1985). Paulus Gerdes further conceptualized this as "frozen mathematics," suggesting that mathematical thinking is often hidden or "frozen" within the techniques of artisans, such as basket weaving or house construction (Gerdes, 1988). While Western mathematics relies on explicit axioms and proofs, the mathematics of the *Saoraja* is implicit, embedded in the physical alignment of pillars and the angular slope of roofs. To the untrained eye, the house is art; to the ethnomathematician, it is a complex geometric problem solved through practical application.

Previous studies in the field of mathematics education have successfully explored the mathematical richness of Indonesian heritage, ranging from Sasaknese architecture (Supiyati et al., 2019) to Buginese culinary traditions (Pathuddin et al., 2021). Furthermore, Prahmana and D'Ambrosio (2020) emphasized that ethnomathematics is not just about forms but about the values embedded in geometric patterns. Building on these foundations, this study extends the scope by utilizing 3D technology to analyze the structural geometry of the *Saoraja La Tenri Bali*. **To clarify the boundaries of our contribution, t**his study builds upon the mathematical analysis established by Ja'faruddin et al. (2025), which formally validated the affine properties of the Saoraja La Tenri Bali. ~~While the previous study focused on the geometric modeling of the artifact itself, the current research shifts the focus to the pedagogical implementation of these findings.~~ **While that prior work focused exclusively**

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on the geometric modeling of the artifact itself (the mathematical 'what'), the current research is an independent mathematics education study focused entirely on pedagogical implementation (the educational 'how'). The central pedagogical novelty of this paper lies in the use of error-based digital ethnomodeling to support conceptual understanding of affine transformations in undergraduate linear algebra. We investigate how undergraduate students, when tasked with reconstructing this heritage site, navigate the conceptual transition from symbolic linear algebra to structural geometric reasoning. However, a significant challenge in ethnomathematics research has been effectively translating these 3D construction techniques into the formal language of academic mathematics. Previous studies have largely focused on 2D symmetry patterns. There remains a gap in understanding the three-dimensional geometric logic—specifically the structural transformations—that govern traditional architecture. How did traditional builders understand the concepts of scaling (to denote hierarchy) or shearing (to ensure stability) without formal theory?

1.1 The Context: Preserving Indonesian Local Wisdom

In the context of Indonesian mathematics education, there is a growing national emphasis on integrating local wisdom (*kearifan lokal*) into the curriculum to strengthen students' character and cultural identity. However, most existing resources focus on primary school levels or simple 2D patterns, such as batik or weaving. There is a scarcity of literature linking complex Indonesian architectural heritage—such as the Bugis *Saoraja*—to advanced university-level mathematics.

1.2 Proposed Approach and Research Objectives

To address this gap, this study proposes a novel approach to "unfreeze" the mathematics of the *Saoraja La Tenri Bali* by integrating "Simple House Theory" (Chen; Ja'faruddin, 2021) with modern digital reconstruction tools. By utilizing Blender, an open-source 3D creation suite, we do not merely replicate the house visually; we reconstruct its geometric logic. This process reveals that the construction of the *Saoraja* implicitly utilizes Affine Transformations—specifically translation, rotation, scaling, and shearing.

We utilize the *Saoraja La Tenri Bali* in Wajo Regency not just as a cultural object, but as a pedagogical context for teaching Affine Transformations. By employing Blender to

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decode this South Sulawesi landmark, this study addresses the following research questions:

(RQ1) How can the geometric analysis of Bugis architecture enhance undergraduate students' understanding of matrix transformations?

(RQ2) How does digital reconstruction serve as a medium to validate indigenous mathematical proficiency?

By answering these questions, this paper seeks to contribute to the broader discourse on ethnomathematics and its pedagogical action, demonstrating that the preservation of Indonesia's architectural heritage and the advancement of STEM education can go hand in hand. This approach resonates with Rosa and Orey (2021), who argue that integrating ethnomathematics into STEM education creates a "glocalized" pedagogy that connects local cultural practices with global scientific knowledge.

2 Theoretical Framework

2.1 Ethnomathematics and Frozen Mathematics

The theoretical foundation of this research lies in D'Ambrosio's Ethnomathematics Program, which seeks to recognize the mathematical practices of identifiable cultural groups. Within this framework, we adopt Gerdes' methodology of "defrosting" hidden mathematical ideas. The *Saoraja* is treated not as a static object, but as a dynamic result of algorithmic thought processes—measuring, aligning, and balancing—that are isomorphic to formal geometric operations. This perspective shifts the narrative from "primitive construction" to "implicit engineering," validating the intellectual depth of indigenous artisans. Such a shift is critical, as noted by Albanese and Perales (2020), for reshaping mathematical conceptions from an ethnomathematical perspective, allowing learners to recognize the epistemological value of diverse cultural forms.

However, it is crucial to clarify our epistemological stance regarding the connection between indigenous knowledge and formal university mathematics. We do not claim that the Bugis master builders implicitly utilized formal affine logic or matrix algebra in their historical practice. Rather, affine transformations provide a highly productive modern academic lens for interpreting and translating the structural algorithms embedded in the artifact. The goal of this ethnomodeling approach is not to

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assert strict historical equivalence, but to establish a dialogue between the two systems, using digital reconstruction to decode cultural heritage through formal geometric language without romanticizing the indigenous knowledge.

2.2 Simple House Theory

We utilize the "Simple House Theory" (Chen; Ja'faruddin, 2021) as our primary analytical lens. This theory abstracts complex traditional houses into geometric primitives (such as prisms and pyramids) and assigns "Building Numbers" based on their projective properties. This framework allows us to systematically categorize the components of the *Saoraja* into three distinct domains: the Substructure (pillars/foundation), the Body (living area), and the Superstructure (roof). This decomposition facilitates a structured mathematical analysis, enabling us to map specific architectural features to their corresponding geometric transformations.

3 Methodology

3.1 Research Design: Qualitative Ethnomodeling with Design-Oriented Task Analysis

This study adopts a **qualitative research design grounded in ethnomodeling**, with a specific focus on **design-oriented task analysis**. Following Rosa and Orey (2011), ethnomodeling is understood as a pedagogical action that translates locally situated cultural practices into formal mathematical representations. **In this context, we utilize ethnomodeling not merely as a theoretical lens, but as a structural methodological procedure. It provides a systematic pathway to translate the *emic* (local/indigenous) architectural practices of the Bugis builders into an *etic* (global/academic) mathematical model using digital tools.** In this research, the translation process is mediated through **digital reconstruction**, where the cultural artifact serves as the epistemic source and the modeling environment functions as a constraint-based interpretive medium.

Rather than evaluating learning outcomes through pre- and post-testing, the study investigates **how mathematical meanings emerge during students' engagement with culturally grounded modeling tasks**. The analytical focus is therefore placed on students' transformation choices, modeling errors, and revision trajectories as indicators of conceptual understanding. This method enables aligns with prior qualitative studies in ethnomathematics

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that emphasize meaning-making processes over performance measures (Albanese; Perales, 2020; Rosa; Orey, 2021).

The research design is **design-oriented** in the sense that the learning task—reconstructing the Saoraja La Tenri Bali under geometric constraints—was intentionally structured to foreground specific affine transformations. The task itself functioned as both an instructional intervention and an analytical lens through which students' mathematical reasoning could be examined.

3.2 Context and Participants

The study was conducted within a cross-national undergraduate geometry course involving 36 students: 16 students from the Department of Smart Computing and Applied Mathematics at **Tunghai University (Taiwan)**, and 20 students from the Department of Mathematics at **Universitas Negeri Makassar (Indonesia)**. All participants had completed introductory linear algebra courses and were familiar with matrix representations of geometric transformations but had limited experience applying these concepts to three-dimensional or culturally situated contexts.

The learning activity was implemented as a four-week collaborative module. Students were organized into cross-national groups of four to five members (typically combining two Taiwanese students with two to three Indonesian students). This grouping strategy was deliberately designed to foster a complementary learning environment where technical programming skills and cultural contextual knowledge could be actively exchanged.

The Saoraja La Tenri Bali, located in the Atakkae Traditional House Complex in Wajo Regency, was selected as the focal artifact due to its geometric complexity, cultural significance, and documented architectural regularities. Its structure provided a mathematically rich environment for exploring affine transformations under authentic structural constraints.

3.3 Learning Task and Data Sources

Data were generated through a multi-phase learning task centered on the **digital reconstruction of the Saoraja** using Blender software. Blender was selected not as a visualization tool, but as a **constraint-based modeling environment** in which geometric

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transformations must be explicitly defined and parameterized. This requirement made students' mathematical assumptions visible through their modeling actions.

The learning task required students to:

1. Decompose the Saoraja into geometric primitives using the Simple House Theory framework (Chen; Ja'faruddin, 2021);
2. Identify projective relationships and "Building Numbers" associated with structural components;
3. Apply affine transformations (translation, rotation, scaling, and shear) to reconstruct the substructure, body, and superstructure of the house under alignment and stability constraints.

During the modeling process, students were provided with semi-structured worksheets that prompted them to explicitly document the transformation matrices and parameters they used for each architectural component before executing them in the software. Instructor interventions were strictly limited to facilitating geometric inquiry. When students encountered structural misalignments in Blender, instructors avoided providing direct procedural solutions; instead, they asked guiding questions (e.g., "What properties of the shape are preserved in your current matrix?" or "How does the physical wood behave at the joint?") to prompt conceptual reflection.

Data sources included:

- Iterative versions of students' Blender models;
- Transformation parameters and matrix configurations used during modeling;
- Group discussion records and reflective learning logs;

Instructor field notes documenting recurring errors and conceptual impasses.

3.4 Analytical Procedure: Digital Hermeneutic Cycle

Data analysis followed an iterative **Digital Hermeneutic Cycle**, adapted from interpretive methodologies in ethnomathematics and design research. The cycle consisted of four recursive phases:

1. **Observation:** Examining the photographic evidence of physical structure.

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2. **Geometric Hypothesis:** Formulation of conjectures regarding the required affine transformations (e.g., hypothesizing shear rather than rotation for slanted roof supports).
3. **Reconstruction:** Implementing the hypothesis in Blender using specific matrix parameters.
4. **Verification:** Comparison between the digital model and the physical artifact. Misalignments were treated as indicators of conceptual inconsistency, prompting revision of the underlying transformation logic.

To establish credibility and methodological rigor, the analysis employed data triangulation. We cross-referenced the formal mathematical parameters embedded in students' submitted Blender files with their qualitative explanations in the reflective logs and the instructor field notes. The reflective logs and discussion records were analyzed using thematic coding procedures. Initial open coding was applied to identify recurring modeling strategies and specific geometric impasses. These codes were subsequently categorized into broader conceptual themes, most notably the systematic transition from "rotation-based rigid body reasoning" to "shear-based structural reasoning." This explicit coding protocol ensures that our claims regarding students' conceptual refinement are empirically grounded in triangulated data, rather than isolated observations.

This analytical process ensured that interpretations were grounded not in visual resemblance alone, but in **mathematically constrained reconstruction**. Students' repeated transitions between rotation- and shear-based models were analyzed as instances of cognitive conflict and conceptual refinement, rather than as technical mistakes.

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Figure 1a (Left) Figure 1b (Right)

Figure 1. Visual comparison between the physical Saoraja La Tenri Bali (Left) and the digitally reconstructed model in Blender (Right), demonstrating geometric fidelity in pillar alignment and roof hierarchy.

Source: Prepared by the authors (2026).

4 Mathematical Analysis of the Artifact

4.1 The Algorithm of the 101 Pillars: Translation and Array Logic

The most distinctive feature of the *Saoraja La Tenri Bali* is its 101 pillars. From a geometric perspective, the placement of these pillars reveals an implicit understanding of **Translation Matrices** and **Grid Algorithms**. As illustrated in **Figure 1** and **Figure 2**, the pillars are arranged in a strict grid pattern. In our digital reconstruction process using Blender, generating these pillars corresponds to an "Array" operation, which is mathematically equivalent to iterative translation.

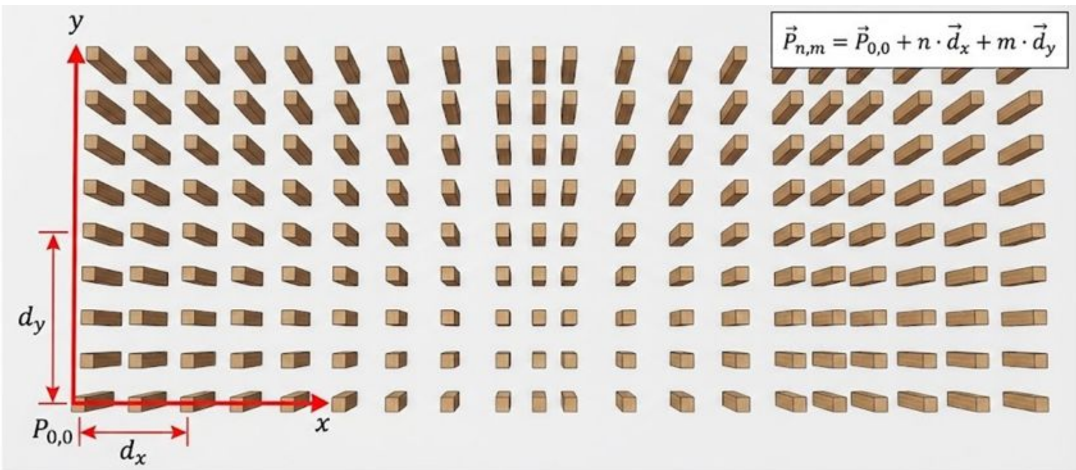

Figure 2. Geometric analysis of the substructure. The arrangement of the 101 pillars follows a strict translational symmetry, modeled mathematically as a 2D array transformation on the xy-plane.

Source: Prepared by the authors (2026).

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If the position of the first pillar (origin) is defined as $P_0 = [x,y,z,1]^T$, the position of the n-th pillar in a row is defined by applying a translation matrix T. The cultural rule that "all pillars must be equidistant" is the physical execution of the following linear operation:

$$P_n = (T_x)^n \cdot P_0 = \begin{bmatrix} 1 & 0 & 0 & n \cdot d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

where d represents the standard distance unit (often based on the *depa* anthropometric unit). This demonstrates that the Bugis builders possessed an algorithmic thinking process akin to a "for-loop" in computer programming.

4.2.1 Shear Transformation: The Geometry of Stability

A critical discovery was the necessity of **Shearing** to accurately model the roof eaves and leaning supports. In standard Euclidean geometry, rectangles remain rectangles. However, in *Saoraja*, the roof sections slant to facilitate drainage and aerodynamics.

During the modeling process, we found that simply rotating a vertical beam caused gaps in the joinery. To maintain the base connection while slanting the structure, a **Shear Matrix** was required. Transforming a vertical beam into a slanted support involves a shear factor $k = \tan\theta$, where θ represents the angle of inclination:

$$Sh_{yz} = \begin{bmatrix} 1 & 0 & k & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This mathematical model explains why the structure remains stable: unlike rotation, shearing preserves the volume and the parallel nature of the support beams (see **Figure 3**). This demonstrates that Shear Transformation is not just an abstract concept, but a structural necessity encoded in the artifact.

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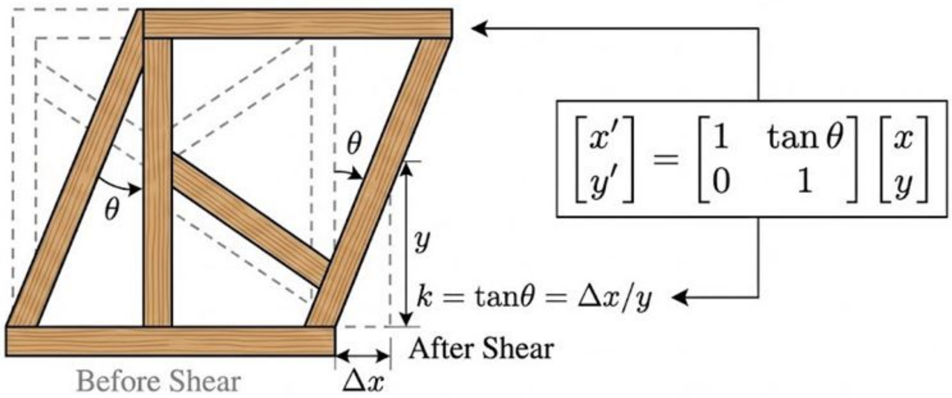

Figure 3. Application of the Shear Transformation (*Sh*). The sloping eaves are modeled not by rotation alone but by shearing a rectangular prism, visualizing the structural optimization for rain drainage ($k = \tan\theta$).

Source: Prepared by the authors (2026).

4.2.2 Scaling: Hierarchy and Anthropometry

The hierarchy of Bugis society is encoded through **Scaling**. The pillars supporting the main noble area (*ale bola*) are significantly larger in diameter than those of the terrace (*lego-lego*). In linear algebra, this is a non-uniform scaling operation. Our measurements show that the scaling factors S_x and S_y (thickness) are functions of the social status associated with the room, while S_z (height) remains constant. This effectively maps "Social Rank" to "Geometric Volume," verifying the principle of **Simple House Theory** where geometric dimensions are semiotic carriers of cultural meaning.

5. Results and Discussion: Pedagogical Outcomes

This study situates the digital reconstruction of the *Saoraja La Tenri Bali* as a pedagogical context for undergraduate linear algebra. Unlike previous studies that focused solely on the geometric modeling of the artifact itself (e.g., Ja'faruddin et al., 2025), this research investigates the **student learning trajectory**—specifically, how the act of modeling mediates the transition from symbolic manipulation to structural reasoning, and how cross-national collaboration enhances cultural appreciation.

5.1 Error Pattern Analysis: From Rotation-Based Modeling to Structural Shear Reasoning

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This section reframes students' initial modeling difficulties not as isolated mistakes, but as **systematic error patterns** that reveal underlying conceptions of affine transformations. Within the digital reconstruction task, students' choices of transformation operations served as externalized representations of their geometric reasoning.

Analysis of the first complete modeling attempts revealed a dominant error pattern: **31 out of 36 students (approximately 85%) initially attempted to model the slanted roof supports (Timpa Laja) using rotation matrices**. This strategy was mathematically plausible from a two-dimensional perspective, as rotation preserves angles and overall shape. However, when implemented in the three-dimensional modeling environment, this approach consistently produced structural inconsistencies. ~~Specifically, rotated beams failed to maintain alignment with their base supports, resulting in visible gaps between the pillar bases and the floor plane.~~ **This visual-first strategy and its subsequent failure were clearly documented in students' reflective logs. For instance, one Taiwanese student (Student T4) noted:**

"I measured the angle from the photo and applied a rotation matrix. Visually, it looked correct from the side, but when I zoomed in, the base of the pillar was floating off the floor. I realized that rotation preserves the shape but ruins the structural connection."

This recurring misalignment functioned as a **diagnostic indicator** of a deeper conceptual issue: students were treating the roof support as rigid bodies rather than as structural elements subject to boundary constraints. In other words, rotation was selected based on visual similarity, not on an analysis of geometric invariants. The Blender environment made this discrepancy immediately perceptible.~~, as the software enforces explicit transformation parameters and does not allow implicit deformation.~~

~~The subsequent shift from rotation to shear emerged through iterative error correction rather than direct instruction.~~

As students attempted to resolve the misalignment, peer discussions and model revisions led them to identify shear as ~~the only transformation~~ **a structurally viable transformation** capable of satisfying two simultaneous constraints: (1) preserving the base position of the support and (2) producing the required inclination of the upper structure. **An Indonesian student (Student I12) reflected on this conceptual shift:**

"We realized we couldn't just rotate it because a real wooden pillar has to sit flat on

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the floor beam. We needed a matrix that slants the top but keeps the bottom fixed (y = 0). Before this, shear was just a matrix to memorize; now I see it is the only way to build a roof without it sliding off."

In formal terms, students moved from applying transformations that preserve congruence to those that preserve parallelism while allowing directional displacement.

This transition represents a **conceptual refinement rather than a procedural correction**. Students did not merely replace one matrix with another; they reconceptualized the functional role of affine transformations in structural contexts. As one Taiwanese student reflected, rotation "looked correct" until examined structurally, while shear emerged as "the only transformation that keeps the house standing." Similarly, Indonesian students connected this realization to traditional construction logic, noting that roof elements must lean without sliding from their foundations.

From an analytical perspective, the rotation-to-shear shift constitutes an **error pattern with pedagogical significance**. The initial error revealed students' reliance on visual heuristics, while the resolution process supported the development of structural geometric reasoning. The modeling task thus transformed shear from an abstract matrix operation into a necessity dictated by architectural constraints, aligning symbolic linear algebra with embodied spatial logic. **This conceptual refinement—moving from the manipulation of abstract symbols to reasoning about structurally constrained spatial movements—highlights the critical role of embodiment in mathematics learning. As Khatin-Zadeh, Eskandari, and Yazdani-Fazlabadi (2024) have demonstrated, mathematical understanding, particularly in domains involving vectors and spatial relationships, is not purely symbolic but can be significantly supported by embodied and gesture-based processes. In our study, the digital ethnomodeling task effectively functioned as an embodied bridge. It allowed students to ground abstract linear algebra (affine transformations) in spatially meaningful, structural actions, demonstrating how culturally situated modeling can mediate advanced mathematical cognition.**

5.2 Cross-National Synergy: Bridging Technical and Cultural Knowledge

The collaborative learning process revealed a complementary distribution of epistemic roles between the two student cohorts.

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- **Cultural Scaffolding by Indonesian Students:** The Indonesian students acted as "cultural experts." When Taiwanese students attempted to scale the pillars uniformly, Indonesian students intervened, explaining that the *Saoraja* follows a strict hierarchy—pillars for the noble area (*Ale Bola*) must be thicker than those for the terrace (*Lego-lego*). This prompted the group to apply **Non-Uniform Scaling** matrices, linking geometric volume to social rank.
- **Technical Scaffolding by Taiwanese Students:** Conversely, Taiwanese students, who had more exposure to coding-based modeling, assisted in translating these cultural rules into Python scripts within Blender. They helped formulate the "array algorithms" to generate the 101 pillars efficiently.
-

This interaction created a **"Third Space"** of learning. A joint reflection from Group 3 illustrated this synergy:

"The Indonesian teammates told us 'why' the pillars had to be that way (the rules of the ancestors), and we (Taiwanese students) figured out 'how' to write the matrix code to make it happen. We weren't just modeling a house; we were translating culture into code together."

5.3 Reframing Mathematical Authority

An additional pedagogical implication concerns students' perception of mathematical authority. By reconstructing the *Saoraja* using formal matrix transformations, students recognized that sophisticated geometric reasoning was embedded in indigenous construction practices, despite the absence of formal notation. This realization reframed mathematics not as an exclusively Western or textbook-based discipline, but as a system of reasoning emerging from practical problem-solving within specific cultural contexts.

Importantly, this shift was not presented as a sociological claim, but as a pedagogical outcome: students ~~validated indigenous mathematical proficiency~~ **recognized the sophistication of indigenous geometric reasoning** through their own formal reconstructions. In doing so, they engaged simultaneously with advanced linear algebra and with culturally grounded knowledge systems, fulfilling the dual objectives of conceptual learning and culturally responsive mathematics education.

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5.4 Limitations and Transferability

While our findings reveal the pedagogical potential of digital ethnomodeling in undergraduate linear algebra, several limitations must be acknowledged. First, the research was conducted with a relatively small sample within a specific cross-national collaborative context. The findings therefore do not aim at statistical generalization, but at **analytical generalization** within qualitative mathematics education research.

Second, the study relied on Blender as a constraint-based modeling environment. Although Blender effectively externalized students' transformation choices, the observed learning trajectories are partially shaped by the affordances and limitations of the software. Other modeling platforms with different constraint systems may mediate students' reasoning in distinct ways. Consequently, the pedagogical outcomes reported here should be understood as emerging from the **interaction between task design, digital medium, and cultural artifact**, rather than from the artifact alone.

Regarding transferability, the contribution of this study does not lie in the specific geometry of the Saoraja La Tenri Bali, but in the **task structure and analytical approach**. The design principles employed—culturally grounded artifacts, explicit transformation constraints, and iterative error-based refinement—can be adapted to other architectural traditions and mathematical topics involving affine or projective transformations. In this sense, the study offers a transferable model for integrating ethnomathematics with advanced mathematical concepts through digital reconstruction, while remaining sensitive to local cultural knowledge systems.

6. Conclusion

This study confirms that the *Saoraja La Tenri Bali* acts as a geometric archive. Through the lens of **Simple House Theory** and digital reconstruction, we decoded the implicit logic of Affine Transformations—Translation, Rotation, Scaling, and Shearing—embedded in its construction. The "101 pillars" are a rigorous execution of translational symmetry, and the sloped eaves are optimized forms described by shear matrices.

We propose that 3D modeling software like Blender serves as a vital **hermeneutic tool** for Digital Ethnomathematics. It allows researchers and educators to "unfreeze" the mathematics hidden in heritage sites, translating embodied craft into explicit linear algebra. Future research should expand this approach to other Austronesian architectural traditions, preserving not

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merely the visual image of these structures, but the sophisticated geometric logic of their creators.

Ultimately, the core educational contribution of this paper lies in demonstrating that culturally grounded, constraint-based modeling tasks can help undergraduate students distinguish among affine transformations through structurally meaningful problem-solving, rather than through routine symbolic exercises alone. By foregrounding the geometric consequences of algebraic operations, such as the structural necessity of shear over rotation, we bridge the gap between abstract mathematics and tangible cultural heritage, offering a pathway for a more inclusive and structurally grounded mathematics curriculum.

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Change Statement Letter

Manuscript ID: BOLEMA-2026-0069

Title: Decoding the Geometry of Heritage: Unveiling Affine Transformations in the Saoraja La Tenri Bali through Digital Reconstruction

Journal: Bolema: Boletim de Educação Matemática

Dear Editor-in-Chief Dr. Danyal Farsani and the Associate Editor,

We sincerely thank the editors and the two reviewers for their constructive, insightful, and highly encouraging feedback. We are grateful for the opportunity to revise and improve our manuscript. We have carefully considered all comments and have made substantial revisions to the manuscript, particularly regarding methodological transparency, evidentiary grounding, and theoretical depth.

Marking of Changes and Additional Revisions:

For the convenience of the reviewers, all significant additions and revisions in the revised manuscript are highlighted in **Bold Blue**, while original sentences that have been removed are indicated with **Green Strikethrough**.

Additionally, we have proactively streamlined both the English Abstract and the Portuguese Resumo to strictly comply with the journal's 250-word limit. We also performed a final proofread to remove four references from the bibliography that were no longer cited in the revised text, ensuring a strict one-to-one correspondence between in-text citations and the reference list.

Below, we provide a point-by-point response detailing how each comment was addressed in the revised manuscript.

Response to Reviewer 1

Reviewer 1, Comment 1: *In this context, it is recommend the use of ethnomodelling that can help the authors in the development of methodological procedures.*

Response: We fully agree. In the revised manuscript, we have expanded **Section**

3.1 to explicitly position ethnomodeling not merely as a theoretical lens, but as a systematic *methodological procedure*. It now clearly describes how ethnomodeling acts as the structural pathway to translate *emic* practices into *etic*mathematical models using digital constraints.

Reviewer 1, Comment 2: *The qualitative design lacks explicit coding procedures, credibility strategies, and detailed data analysis protocols.*

Response: Thank you for pointing out this critical methodological gap. We have significantly overhauled **Section 3.4 (now titled "Analytical Procedure: Digital Hermeneutic Cycle and Data Triangulation")**. We added explicit protocols detailing how we achieved credibility through data triangulation (cross-referencing Blender files, learning logs, and field notes). Furthermore, we explicitly outlined our thematic coding procedures, explaining how initial open codes regarding modeling impasses were categorized into the broader conceptual theme of "rotation-to-shear transition."

Reviewer 1, Comment 3: *Claims about conceptual learning are not sufficiently supported by thick empirical evidence such as model excerpts or extended student episodes.*

Response: We have addressed this by enriching **Section 5.1** with thick empirical evidence. We included specific, direct quotes from students' reflective logs (e.g., Student T4 from Taiwan and Student I12 from Indonesia) that explicitly illustrate their cognitive conflict and the structural reasoning that led them to abandon rotation in favor of shear.

Reviewer 1, Comment 4: *Some mathematical explanations require greater precision, especially regarding invariants and properties of shear transformations.*

Response: We have enhanced the mathematical precision in both **Section 4.2.1** and **Section 5.1**. We added explicit language regarding geometric invariants, specifically noting that shear is the required transformation because it "preserves parallelism while allowing directional displacement" and maintains the invariant baseline ($\boldsymbol{y} = \mathbf{0}$), fulfilling the structural boundary constraints.

Reviewer 1, Comment 5: *Cultural grounding would benefit from clearer engagement* *with community knowledge sources.*

Response: To strengthen the cultural grounding, we added details in **Section 3.2 and 5.2** describing how the cross-national grouping deliberately functioned to provide cultural scaffolding. Indonesian students acted as cultural experts (community knowledge proxies) to guide the technical modeling performed collaboratively with Taiwanese students. Additionally, we added an epistemological clarification in **Section 2.1** to ensure we are translating, rather than romanticizing, this knowledge.

Response to Reviewer 2

Reviewer 2, Comment 1: *The paper should sharpen the central contribution... state more explicitly that the pedagogical novelty lies in the use of error-based digital ethnomodeling to support conceptual understanding of affine transformations... foregrounded earlier and carried consistently through the abstract, introduction, and conclusion.*

Response: We deeply appreciate this structural advice. We have revised the **Abstract**, **Introduction**, and **Conclusion** to explicitly foreground this central pedagogical novelty. The focus on "error-based digital ethnomodeling" is now the consistent golden thread tying the manuscript together.

Reviewer 2, Comment 2: *The methodology needs a little more detail... how long did the activity last, how were the groups formed, what prompts or worksheets were used, what instructor interventions were allowed, and how were the reflective logs analyzed?*

Response: We have provided comprehensive procedural transparency. In **Section 3.2**, we specified that the module lasted four weeks and described the cross-national group formations. In **Section 3.3**, we detailed the use of semi-structured worksheets and clarified that instructor interventions were strictly non-procedural, focusing instead on geometric inquiry. Log analysis procedures were added to **Section 3.4**.

Reviewer 2, Comment 3: *Clarify the relationship between the present paper and Ja'faruddin et al. (2025)... A short paragraph clarifying what is newly added here would strengthen the article considerably.*

Response: We have added a dedicated paragraph in the **Introduction** that establishes a clear boundary between the two works. We explicitly state that while the 2025 paper focused on the formal geometric validation of the artifact (the mathematical 'what'), the current manuscript is an independent mathematics education study focused entirely on pedagogical implementation and student learning (the educational 'how').

Reviewer 2, Comment 4: *Results and Discussion section would benefit from a clearer separation between mathematical findings about the artifact and pedagogical findings about student learning.*

Response: We have clarified this boundary by updating the section structure. **Section 4** strictly establishes the mathematical foundations of the ethnomodel, while **Section 5 (Pedagogical Outcomes)** is now exclusively dedicated to analyzing how students engaged with the model, errors, and cultural synergies.

Reviewer 2, Comment 5: *Provide one or two more concrete excerpts from student reflections or discussions to support the claim that this was a conceptual shift rather than a mere software correction.*

Response: As suggested, we have inserted concrete excerpts from student reflective logs in **Section 5.1**. These quotes demonstrate that the shift from rotation to shear was indeed recognized by the students as a structural necessity rather than just a software fix.

Reviewer 2, Comment 6: *Engage more explicitly with embodiment in mathematics learning... This would be an excellent place to cite Khatin-Zadeh, Eskandari, and Yazdani-Fazlabadi (2024).*

Response: This is a brilliant suggestion that adds significant depth to our analysis. We have added a new paragraph at the end of **Section 5.1** explicitly framing the students' transition from symbolic to spatial reasoning as an embodied process. We have incorporated and cited the recommended work by Khatin-Zadeh et al. (2024) to support the argument that digital ethnomodeling serves as an embodied bridge for abstract algebra.

Reviewer 2, Comment 7: *Theoretical framework could say a little more about how ethnomathematics and formal university mathematics are being connected* *epistemologically...*

Response: We have added an epistemological clarification at the end of **Section 2.1**. We explicitly state that we do not claim historical equivalence (i.e., that Bugis builders used written matrices). Instead, we clarify that affine transformations provide a "productive modern academic lens" to translate and dialogue with indigenous structural algorithms, strictly avoiding romanticization.

Reviewer 2, Comment 8: *Be careful with some strong claims about validation... recommend slightly moderating the phrasing so that the paper remains analytically precise rather than rhetorically overstated.*

Response: We thoroughly reviewed the manuscript and moderated absolute phrasing. For example, in **Section 5.1**, "the only transformation" was changed to "a structurally viable transformation," and in **Section 5.3**, "validated indigenous mathematical proficiency" was revised to "recognized the sophistication of indigenous geometric reasoning."

Reviewer 2, Comment 9: *English expression could be made more concise and more natural... reduce repeated use of phrases like "this study demonstrates".*

Response: A comprehensive language edit was conducted throughout the manuscript. We have tightened sentences, improved transitions, and systematically reduced the repetition of phrases such as "this study demonstrates" and "this approach allows" to enhance overall readability and flow.

Reviewer 2, Comment 10: *The conclusion is good but could be made even stronger by returning more directly to the undergraduate mathematics education contribution.*

Response: We entirely rewrote the final paragraph of the **Conclusion**(Section 6). It now directly reiterates the core pedagogical payoff: that constraint-based modeling tasks help students distinguish among affine transformations through structurally meaningful problem-solving, rather than through routine symbolic exercise.

We believe these revisions have significantly strengthened the methodological rigor, empirical depth, and theoretical framing of our study. We thank the reviewers once again for their time and expertise, and we hope the revised manuscript is now suitable for publication in *Bolema*.

Sincerely,

The Authors